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# Age of Information with Collision-Resolution Random Access

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***Abstract*—This paper studies the collision-resolution random access (CRRA) protocol for information update systems with age of information (AoI) requirements. AoI measures the information freshness, defined as the time elapsed since the generation of the last successfully received update. When a large number of users contend and send update packets to a common access point (AP), conventional random access protocols such as slotted Aloha (SA) simply ignore the collisions and inform the users to contend for the wireless channel again, thus leading to high average AoI due to the high collision probability. This paper argues that resolving collisions can improve the information freshness. When a collision occurs in CRRA, only the collided users enter a collision resolution procedure (CRP) to contend and access the channel until all of their update packets are received successfully. In particular, the time to receive a user's update packet in the CRP is random, which complicates the AoI analysis. This paper theoretically analyze the average AoI of CRRA. Simulations show that CRRA significantly reduces the average AoI of SA, especially when update packets have a large payload. Furthermore, CRRA is more robust against the estimation error of the number of random-access users, and it is thus a promising solution to networks with time-varying traffic.**

***Index Terms*—Age of information, collision resolution, information freshness, random access**

## I. Introduction

In recent years, age of information (AoI) has been regarded as a key performance metric to measure information freshness in various Internet-of-Things (IoT) applications [1], [2]. Such information update systems are expected to receive fresh updates from a massive number of IoT devices. Consider a common information update scenario where multiple IoT devices send status updates to the same access point (AP), e.g., in environmental and agricultural machine-type communications (MTC). The AoI of an IoT device, measured at the AP, is defined as the elapsed time since the generation of the latest received update, i.e., the age of the newest received update [1]. Prior work showed that maximizing throughput or minimizing delay does not necessarily guarantee a low AoI [2].

In large-scale wireless IoT networks, age-optimal scheduling problems have been investigated to achieve low network-wide average AoI [2], [3]. In particular, AoI optimization has been explored in both scheduled access [4], [5] and random access networks [6]–[9]. Scheduled access generally requires centralized coordination and decision, which may be restrictive for many IoT scenarios with a large number of devices. Hence, efficient random access protocols operated in a distributed and decentralized way are promising, e.g., slotted Aloha (SA)-like random access protocols have been regarded as an effective solution to achieve low average AoI [10]. In [5], each SA user can send its update packet with a fixed probability that can be optimized ahead of time. Different SA-based protocols are proposed to achieve high information freshness, e.g., [8] proposes to leverage the instantaneous AoI to adjust the transmission probability, and [9] allows up to a certain number of packet retransmissions to improve the reliability of packet reception. Despite these efforts, conventional AoI-aware SA-based protocols do not deal with interference or collision when more than one user sends simultaneously to the AP. For example, in [5], the AP simply ignores the collision and informs the users to contend the channel again, thus leading to a high average AoI due to the high collision probability at every time slot.

The splitting or the collision-resolution algorithms have long been proposed and investigated in the literature [11], [12]. In particular, users involved in a collision are split and resolved recursively, e.g., a collision with a large number of users is gradually split into several collisions with a smaller number of users. When only one user is left, the process moves backwards and recursively resolves previous collisions, thus the number of contending users goes on decreasing [12]. While the collision-resolution algorithms have been shown to improve the random-access throughput over SA [12], its performance on AoI has not been investigated: how to incorporate with the new AoI requirement remains to be explored.

For the first time, this paper investigates the collision-resolution random access (CRRA) protocol for information update systems. Unlike the conventional collision-resolution protocol designed for high throughput without packet loss, we design the new CRRA protocol targeted for low average AoI. Specifically, we allow and design packet drop to improve information freshness, since new packets always contain the latest information. A key challenge in the theoretical analysis of average AoI is that the time to receive an update packet of a user is random during the collision resolution procedure,



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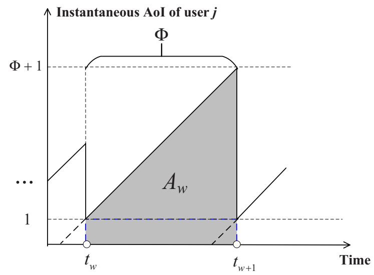


Fig. 1: An example of the instantaneous AoI of user $j$ in SA. The $w$-th and $(w+1)$-th updates occur at times $t_w$ and $t_{w+1}$, respectively, so the instantaneous AoI drops to 1. The duration between the two consecutive updates is $\Phi$.

e.g., the instantaneous AoI drop is random upon a successful update. To this end, we dig into the instantaneous AoI evolution carefully and derive the theoretical average AoI of CRRA, which provides insights for designing algorithms with practical limitations.

We validate our theoretical analysis by simulations. Simulations show that CRRA outperforms SA significantly in average AoI. By adjusting the update probability, the optimal average AoI of CRRA is reduced by 18% compared with SA when the number of users is 50 and the payload size of update packets is 256 bytes. Furthermore, in networks with time-varying traffic, CRRA provides a more stable average AoI since it is more robust against the estimation error of the number of random-access users.

## II. Preliminaries

We study a time-slotted system with an AP and $N$ users. The AP wants to receive information from the users as fresh as possible, and AoI is used to measure information freshness. Specifically, at time instant $t$, the instantaneous AoI of user $j$, denoted by $\Delta_j(t)$, is defined by $\Delta_j(t) = t - G_j(t)$, where $G_j(t)$ is the generation time of the latest update packet received by the AP from user $j$ [1], [2]. A lower $\Delta_j(t)$ means that the AP has fresher information of user $j$ at time $t$.

Average AoI [1], [2], defined as the time average of the instantaneous AoI, is usually adopted to measure information freshness over a long period. The average AoI of user $j$ is

$$\bar{\Delta}_j = \lim_{T\to\infty} \frac{1}{T}\int_0^T \Delta_j(t)dt. \tag{1}$$

This paper considers the *generate-at-will* model [2] where the information about the observed phenomena can be sampled by the user (say, a sensor), and the update packet can be generated at any time of its own choice, e.g., when the user has the transmission opportunity. In other words, the user can decide when to sample information and instantaneously trigger the circuitry to accomplish packet generation and transmission. To realize the generate-at-will model in practice, the communication layer of the user can "pull" a request from the upper application layer just when there is an upcoming transmission opportunity. This ensures that the sampled information is as fresh as possible, e.g., a sensor reading is obtained just before its transmission opportunity. Besides, the user can generate and send an update packet immediately after receiving a request from the AP.

We take slotted Aloha (SA) as an example to show the calculation of the average AoI $\bar{\Delta}_j$. Following the generate-at-will model, we assume that in SA, each user generates and sends a new update packet with probability $p_{SA}$ in each time slot. A practical example of this scenario is that a sensor may generate a time-stamped packet to report its status at the current time slot to monitor physical quantities. An example of the instantaneous AoI of user $j$, $\Delta_j(t)$, is shown in Fig. 1. In general, $\Delta_j(t)$ grows linearly and continuously. In this example, we assume that the AP successfully receives user $j$'s update packets at times $t_w$ and $t_{w+1}$, i.e., two consecutive successful updates. Thus, at times $t_w$ and $t_{w+1}$, $\Delta_j(t)$ drops to 1 (time slot).

This paper adopts the collision model where a packet is successfully sent only if no other packet is sent at the same time. That is, when the AP receives only one packet in a slot, the AP can decode it successfully; if the AP receives packets from multiple users in a slot simultaneously, a collision occurs. To compute the average AoI $\bar{\Delta}_j^{SA}$ for user $j$, let $\lambda$ be the probability that a successful update from user $j$ occurs at the end of a slot, i.e., $\lambda = (1-p_{SA})^{N-1}p_{SA}$. After one successful update, denote by $\Phi$ the number of time slots until the next successful update (see Fig. 1). Then $\Phi$ is a geometric random variable with a probability mass function (PMF)

$$\Pr(\Phi = \phi) = (1-\lambda)^{\phi-1}\lambda,\ \phi = 1, 2, ... \tag{2}$$

i.e., $\phi-1$ slots without successful updates followed by one successful update. Note that $\mathbb{E}[\Phi] = 1/\lambda$ and $\mathbb{E}[\Phi^2] = (2-\lambda)/\lambda^2$, where $\mathbb{E}[\cdot]$ denotes the expectation operation. Therefore, $\bar{\Delta}_j^{SA}$ is computed by using the renewal theory [1]

$$\begin{aligned}\bar{\Delta}_j^{SA} &= \lim_{W\to\infty}\frac{\sum_{w=1}^{W} A_w}{\sum_{w=1}^{W}\Phi_w} = \frac{\mathbb{E}\left[\Phi + \frac{1}{2}(\Phi)^2\right]}{\mathbb{E}\left[\Phi\right]} \\ &= 1 + \frac{\mathbb{E}\left[\Phi^2\right]}{2\mathbb{E}\left[\Phi\right]} = \frac{1}{2} + \frac{1}{p_{SA}(1-p_{SA})^{N-1}},\end{aligned} \tag{3}$$

where $A_w$ is the area between the $w$-th and $(w+1)$-th successful updates (i.e., the shaded area in Fig. 1). It is easy to show that $\bar{\Delta}_j^{SA}$ is minimized when $p_{SA} = 1/N$. In general, SA does not perform well when the number of users is large because it does not deal with the collision. The next section considers CRRA to improve information freshness.

## III. Collision-Resolution Random Access

### A. System Model

In CRRA, time is divided into different access periods for users to access the channel. An access period consists of several time slots. At the beginning of an access period, the AP sends a polling frame to notify all users that a new access period begins. Each user generates and sends a new update packet with access probability $p_{CR}$. Fig. 2 shows an example of CRRA with four users and five access periods.

Let $K$ denote the number of simultaneous transmissions in an access period. We assume that CRRA begins a collision resolution procedure (CRP) when $2 \le K \le K_{\max}$, where

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$$A = BT_f + \frac{1}{2}{T_f}^2 + (B + T_f)D + \frac{1}{2}D^2 + (T_l - D) + \frac{1}{2}(T_l - D)^2 = T_f + T_l + T_f T_l' + \frac{1}{2}T_f^2 + \frac{1}{2}T_l^2 + D^2 - D'D + (T_f D - T_f D') + (T_l' D - T_l D) \tag{4}$$

$$\bar{\Delta}_j = \lim_{W \to \infty} \frac{\sum_{w=1}^{W} A_w}{\sum_{w=1}^{W} (T_f^w + T_l^w)} = \frac{\mathbb{E}[T_f] + \mathbb{E}[T_l] + \mathbb{E}[T_f]\mathbb{E}[T_l] + \frac{1}{2}\mathbb{E}[{T_f}^2] + \frac{1}{2}\mathbb{E}[{T_l}^2] + \mathbb{E}[D^2] - \mathbb{E}[D]^2 + \mathbb{E}[T_l]\mathbb{E}[D] - \mathbb{E}[T_l D]}{\mathbb{E}[T_f] + \mathbb{E}[T_l]} \tag{5}$$

$K_{\max}$ is the maximum number of simultaneous transmissions that leads to a CRP. Note that SA is a special case of CRRA with $K_{\max} = 1$. We consider $K_{\max} = 3$ and the generalization to a larger $K_{\max}$ is straightforward. We remark that the AP knows $K$ only: it does not know which users are included in a specific collision, and therefore a random transmission in the CRP is the most viable option. While it is hard to obtain $K$ in practice, the ideal scenario enables us to investigate the theoretical performance and provide insights for designing algorithms with practical limitations, as will be presented in Section III-C.

Depending on different $K$, an access period may have a different duration (i.e., different numbers of time slots):

- If $K = 0$, no packets are sent from the users; the duration is $T_0 = 1$, e.g., the second access period in Fig. 2;
- If $K = 1$, one user sends successfully and the duration is $T_1 = 1$, e.g., the first access period in Fig. 2;
- If $K > K_{\max}$, no packets can be decoded and the duration of the access period is $T_{K > K_{\max}} = 1$, e.g., the fourth access period in Fig. 2;
- If $2 \leq K \leq K_{\max}$, CRRA enters a CRP when two or three users send simultaneously ($K_{\max} = 3$), as shown in the third and the fifth access period in Fig. 2, respectively. The duration of the access periods are different under different $K$. We use $T_K$ to denote the duration of an access period with a $K$-user CRP.

To explain the CRP, we consider the 5th access period shown in Fig. 2 as an example. In the first time slot of the access period, the AP sends a polling frame, and three users 1, 2, and 3 choose to send an update packet, leading to a collision. In the next time slot, the AP broadcasts a feedback, referred to as a trigger frame, to the users. The trigger frame informs the users who sent a packet in the previous time slot (i.e., users 1, 2, and 3) to enter a $K = 3$ CRP. Users who did not send a packet in the last time slot (i.e., users other than users 1, 2, and 3) will remain silent in the entire CRP, waiting for the next polling frame to enter a new access period. With $K = 3$, the trigger frame issues a transmission probability $p_3$ to users 1, 2, and 3, i.e., the three collided users randomly access the channel and send their update packets with probability $p_3$ in the first time slot of the CRP. In Fig. 2, we see that only user 2 and user 3 send, while user 1 chooses not to send. Notice that to lower the AoI, user 2 and user 3 should drop their old packets and send new packets, since new packets always contain the most up-to-date information. In our proposed CRRA protocol, users always generate and send a new packet when they have an opportunity to access the channel in the CRP, i.e., the generate-at-will model.

Continuing with the example above, the simultaneous transmission from user 2 and user 3 results in a collision again. In the second time slot of the CRP, the AP sends a trigger frame

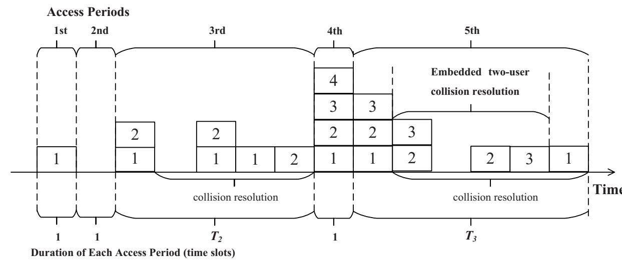


Fig. 2: An illustrative example of access periods with different durations in CRRA.

to the users, indicating that the collision still occurs and now an *embedded* two-user CRP for user 2 and user 3 starts (i.e., user 1 keeps silent). The trigger frame issues a transmission probability $p_2$ to user 2 and user 3 so that they send their new packets again, but now with probability $p_2$. However, in the second time slot of the CRP, neither user 2 nor user 3 chooses to send.

In the subsequent time slot, the AP triggers users 2 and 3 again. Now only user 2 sends and the update is successful. In the fourth time slot of the CRP, the AP triggers user 3 to send. In other words, user 3 sends its new packet with probability $p_1 = 1$ to minimize the access delay, and the embedded two-user CRP ends. Finally, in the fifth time slot of the CRP, the AP triggers the transmission of the last user (user 1). As a result, the 5th access period ends with a duration $T_3 = 6$, as shown in Fig. 2.

### *B. Average AoI of CRRA*

We analyze the average AoI of CRRA by considering the instantaneous AoI evolution of user $j$. Notice that in the following analysis, we use the number of time slots as the unit of the average AoI. As explained above, a time slot in CRRA includes the durations of a polling/trigger frame and an update packet. However, polling/trigger frames are not required in SA. A comparison between CRRA and SA, especially the impact of the polling/trigger frames on the average AoI, will be presented in Section IV.

Let $Z$ denote the number of access periods between two consecutive successful updates. Let $T_f$ and $T_l$ denote the duration of the $f$irst $Z - 1$ access periods and the $l$ast access period, respectively (between two consecutive successful updates). Suppose that immediately after the last successful access period, the instantaneous AoI of user $j$, $\Delta_j$, is $B$. $\Delta_j$ increases linearly until the next successful update. Fig. 3 shows the instantaneous AoI evolution between the two successful updates. Note that it is possible that user $j$ updates successfully before the CRP ends, so the instantaneous AoI first drops to 1 and then continues to increase linearly.

We define $D$ as the duration from the beginning of the successful access period until user $j$ updates successfully. The

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area of $A$ in Fig. 3 is given by (4). Note that $B = 1 + T_l' - D'$, where $T_l'$ and $D'$ are the corresponding parameters in the previous successful access period. The average AoI of CRRA is computed by (5) using the renewal theory, where the index $w$ in (5) denotes the $w$-th update. In the following, we compute the components in (5).

The number of access periods between two consecutive successful updates, $Z$, is a geometric random variable, i.e., user $j$ fails to update in the first $Z-1$ access periods and then successfully updates in the $Z$-th access period. With $K_{\max} = 3$, let $\mu$ denote the probability of a successful update for user $j$ in an access period. Then $\mu$ can be expressed as

$$\mu = p_{CR} \sum_{i=0}^{2} \binom{N-1}{i} (1-p_{CR})^{N-1-i} (p_{CR})^i. \quad (6)$$

That is, to successfully update in an access period, user $j$ may collide with zero, one, or two of the other $N-1$ users. Next, we analyze the duration of the first $Z-1$ access periods, $T_f$, and the last access period, $T_l$.

**The first** $Z-1$ **access periods**: For the first $Z-1$ access periods, there are three possibilities for each access period:

(1) User $j$ sends but collides with more than two users among the other $N-1$ users. Or, when user $j$ does not send a packet, $K=0$, $K=1$ or $K>3$ other users send. The duration of all these access periods is 1;
(2) User $j$ does not send a packet, but two other users send simultaneously, i.e., the access period has a duration of $T_2$ with a two-user CRP;
(3) Same as (2) except that $K=3$, i.e., the access period has a duration of $T_3$ with a three-user CRP.

Given $Z = z$, denote the numbers of the above three scenarios by $\Theta_1$, $\Theta_2$, and $\Theta_3$, respectively. $(\Theta_1, \Theta_2, \Theta_3)$ follows a multinomial distribution with a PMF

$$\begin{aligned} &P_{\Theta_1,\Theta_2,\Theta_3}(\Theta_1=\theta_1, \Theta_2=\theta_2, \Theta_2=\theta_3 | Z=z) \\ &\qquad = \frac{(z-1)!}{\theta_1!\theta_2!\theta_3!} {\varphi_1}^{\theta_1} {\varphi_2}^{\theta_2} {\varphi_3}^{\theta_3}, \quad (7) \\ &\theta_1+\theta_2+\theta_3 = z-1, \\ &\varphi_2 = \binom{N-1}{2} (1-p_{CR})^{N-2} {p_{CR}}^2/(1-\mu), \\ &\varphi_3 = \binom{N-1}{3} (1-p_{CR})^{N-3} {p_{CR}}^3/(1-\mu), \\ &\varphi_1 = 1-\varphi_2-\varphi_3. \end{aligned}$$

The duration $T_f$ is then $T_f = \Theta_1 + \Theta_2 T_2 + \Theta_3 T_3$, and hence we can compute $\mathbb{E}[T_f]$ and $\mathbb{E}[T_f^2]$ accordingly, where the computations of $T_2$ and $T_3$ are detailed as follows.

**The last access period**: Similar to $T_f$, there are three possible scenarios for $T_l$:

(1) If only user $j$ sends a packet, no CRP is required. $T_l = D = 1$ and the instantaneous AoI drops to 1. The probability of this scenario is $\alpha_1 = p_{CR}(1-p_{CR})^{N-1}/\mu$.
(2) If user $j$ and one of the other $N-1$ users send simultaneously, a $K=2$ CRP is requried, i.e., $T_l = T_2$. The probability of this scenario is

$$\alpha_2 = (N-1){p_{CR}}^2(1-p_{CR})^{N-2}/\mu. \quad (8)$$

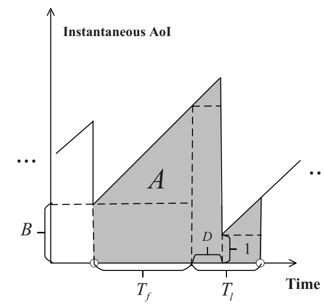


Fig. 3: Possible instantaneous AoI evolution in CRRA.

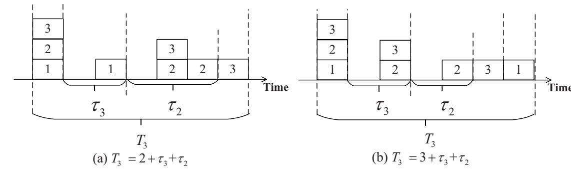


Fig. 4: CRRA collision resolution procedure examples with $K=3$ and different possible $T_3$: (a) $2+\tau_3+\tau_2$; (b) $3+\tau_3+\tau_2$.

The collision can be resolved when a packet from one of the two collided users is received. Let $\tau_2$ denote the time required to receive a packet from one of the two users. Then $\tau_2$ is a geometric random variable with the parameter $\chi_2 = 1-(1-p_2)^2 - {p_2}^2$. Moreover, the remaining user needs one more time slot. Thus, $T_2 = 1 + \tau_2 + 1$ (where the first "1" denotes the time slot that the two users collide at the beginning of the access period, thus requiring a two-user CRP), and $\mathbb{E}[T_2] = 2 + 1/\chi_2$, $\mathbb{E}[(T_2)^2] = 4 + 4/\chi_2 + (2-\chi_2)/(\chi_2)^2$. Notice that user $j$'s update packet may be recovered before the end of the CRP. Hence, $D$ could be

$$D = \begin{cases} 1+\tau_2, & w.\ prob.\ \nu_1 = \frac{p_2(1-p_2)}{\chi_2} = \frac{1}{2} \\ 2+\tau_2, & w.\ prob.\ \nu_2 = \frac{p_2(1-p_2)}{\chi_2} = \frac{1}{2} \end{cases}. \quad (9)$$

(3) If user $j$ and two of the other $N-1$ users send simultaneously, a $K=3$ CRP is required, i.e., $T_l = T_3$. The probability of this scenario is

$$\alpha_3 = \binom{N-1}{2} {p_{CR}}^3 (1-p_{CR})^{N-3}/\mu. \quad (10)$$

Note that there is an embedded two-user CRP in the three-user CRP. As shown in Fig. 4, the embedded two-user CRP may start when (a) a single packet is received (e.g., packet 1 is received and then a two-user CRP for packet 2 and packet 3 starts), and when (b) two packets are sent and collided (e.g., packet 2 and packet 3 collide and then a two-user CRP for them starts; also notice that finally packet 1 is sent). Denote $\tau_3$ as the time required for starting the embedded two-user CRP. Then $\tau_3$ is a geometric random variable with the parameter $\chi_3 = 1-(1-p_3)^3 - (p_3)^3$. Hence, $T_3$ could be

$$T_3 = \begin{cases} 1+\tau_3+\tau_2+1, & w.\ prob.\ \eta_1 = \frac{3p_3(1-p_3)^2}{\chi_3} \\ 1+\tau_3+\tau_2+2, & w.\ prob.\ \eta_2 = \frac{3{p_3}^2(1-p_3)}{\chi_3} \end{cases}. \quad (11)$$

With the probability distribution (11), we can compute $\mathbb{E}[T_3]$ and $\mathbb{E}[(T_3)^2]$. As with $K=2$, user $j$'s update

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packet may be recovered at different times during the CRP. Specifically, $D$ could be

$$D = \begin{cases} 1+\tau_3, & w.\ prob.\ \frac{1}{3}\eta_1 \\ 1+\tau_3+\tau_2, & w.\ prob.\ \frac{2}{3}\eta_1\nu_1+\frac{2}{3}\eta_2\nu_1=\frac{1}{3} \\ 2+\tau_3+\tau_2, & w.\ prob.\ \frac{2}{3}\eta_1\nu_2+\frac{2}{3}\eta_2\nu_2=\frac{1}{3} \\ 3+\tau_3+\tau_2, & w.\ prob.\ \frac{1}{3}\eta_2 \end{cases} . \quad (12)$$

Based on the above three scenarios, the duration $T_l$ is given by $T_l = \alpha_1 + \alpha_2 T_2 + \alpha_3 T_3$, and we can compute $\mathbb{E}[T_l]$ and $\mathbb{E}[T_l^2]$ accordingly. $\mathbb{E}[D]$ and $\mathbb{E}[D^2]$ can also be computed based on the probabilities of the above three scenarios and different $D$ (e.g., (9) and (12)). Finally, with the probability distributions of $D$ and $T_l$, we can compute $\mathbb{E}[T_l D]$.

### C. Practical CRRA Without Knowing $K$

We adopt the practical CRRA protocol without knowing $K$ as proposed in [12] to our AoI scenario. The design of this protocol is based on the fact that it is more likely to have a collision with two users. Compared with the case of known $K$, the practical CRRA protocol without knowing $K$ is modified as follows.

When a collision occurs in an access period, since $K$ is not known, the AP sends a trigger frame and broadcasts a constant transmission probability $q$ in the first time slot of the CRP, regardless of the number of users involved in a collision. The collided users (those who have just transmitted) enter a $K$-user CRP, i.e., they randomly send their update packets with probability $q$. Let $L$ be the number of transmitting users in the first time slot of the CRP:

- If $L = 0$, the $K$ collided users start the $K$-user CRP again in the next time slot.
- If $L = 1$, one user successfully sends an update. The remaining $K - 1$ users transmit with probability 1 in the second slot of the CRP. The CRP ends if the second slot is also a successful transmission. (A collision with two users is highly likely to occur.) Otherwise, the remaining users enter an embedded $K - 1$ CRP.
- If $L \geq 2$, a collision occurs. Then, only the $L$ collided users enter an $L$-user CRP, while the remaining $K - L$ users leave the CRP. This also ensures that the total CRP is relatively short to reduce AoI.

As with a known $K$, users always send a new packet when they have an opportunity to send it during the CRP. The key difference here is that not all users in the CRP can have a successful update [12]. The theoretical analysis of the practical CRRA is much more complicated, e.g., the time to update in the CRP is random, and a new case where a user enters a CRP but fails to update should be considered. However, simulation results below show that such a practical CRRA protocol has almost the same average AoI as the case where $K$ is known.

## IV. CRRA Performance Evaluation

We now verify the theoretical analysis of the average AoI via simulations and compare the performances between CRRA and SA. As mentioned earlier, the time slot duration of CRRA is longer than SA due to polling and trigger frames. Hence, this section uses millisecond ($ms$) as the unit of the average AoI for a fair comparison. Our simulation follows the network parameters defined in the IEEE 802.11 standards [13], as shown in Table I. We first consider a payload of 256 bytes for each update packet in Figs. 5 and 6. The impacts of different payload sizes will be examined in Fig. 7.

TABLE I: Network Simulation Parameters

| | |
|---|---|
| Payload of an update packet (bytes) | 16, 32, 64, 128, 256 |
| Data transmission bitrate (Mbps) | 6 |
| PHY-layer header duration ($\mu$s) | 20 |
| MAC header + PHY pad (bits) | 246 |
| Signal extension time ($\mu$s) | 6 |
| Polling/Trigger Frame (bits) | 160 |

Fig. 5 plots the average AoI versus the access probability with (a) $N = 5$ and (b) $N = 10$ users. The access probability in the $x$-axis is $p_{SA}$ for SA and $p_{CR}$ for CRRA. In CRRA, $p_2$ and $p_3$ are set to 0.5 and 0.41 to minimize $\mathbb{E}[T_2]$ and $\mathbb{E}[T_3]$, respectively.[1] Also, $q = p_2 = 0.5$ for the practical CRRA protocol (note that a collision with two users occurs highly likely). For the theoretical results, we use the above parameters to compute the average AoI of SA and CRRA using the formulas derived in previous sections. For the simulation results, we collect the instantaneous AoI based on a large number of access periods (i.e., a user may or may not have a successful update in each access period), from which we compute the average AoI. As plotted in Fig. 5, the simulation results are consistent with the theoretical analysis. In addition, the average AoI of the practical protocol with an unknown $K$ almost coincides with the $K_{\max} = 3$ case, indicating that the theoretical average AoI analysis with $K_{\max} = 3$ approximates well to the practical CRRA protocol (see also Fig. 6 under different $N$; we therefore omit the discussion between the known and unknown $K$ scenarios in the following).

We see from Fig. 5 that SA achieves the minimum average AoI when $p_{SA} = 1/N$ in both plots, but the minimum average AoI of CRRA is significantly lower than that of SA. Specifically, CRRA achieves the minimum average AoI when $p_{CR}$ is around 0.4 in (a) $N = 5$ and 0.15 in (b) $N = 10$, i.e., CRRA has a larger $p_{CR}$ than $p_{SA}$ to have a minimum average AoI. Moreover, Fig. 6(a) plots the optimal access probability versus the number of users $N$. Given a fixed $N$, we have $p_{CR} > p_{SA}$. Thanks to the CRP, CRRA encourages users to send more aggressively in the access period to lower AoI.

We observe that when CRRA is used, the average AoI is more stable in the same access probability range than that in SA. In real random access scenarios, the number of users $N$ in the system may not be known in advance, e.g., an online estimation of $N$ is required at the AP. Fortunately, CRRA is more tolerant of the estimation errors in $N$. In Fig. 5(b), for example, suppose that the real number of users is 10, but the estimated number is 5, i.e., an estimation error. If $p_{SA} = 0.2$ is used in SA (optimal when $N = 5$), the resulting average AoI is much higher than the optimal average AoI when $p_{SA} = 0.1$ (optimal when $N = 10$). By contrast, if $p_{CR} = 0.4$ is used in CRRA (optimal when $N = 5$), the average AoI only increases little over the optimal value when $p_{CR} = 0.15$ (optimal when

[1] $p_2 = 0.5$ and $p_3 = 0.41$ are found by minimizing $\mathbb{E}[T_2] = 2 + 1/\chi_2$ and $\mathbb{E}[T_3] = 2 + 1/\chi_3 + 1/\chi_2 + \eta_2$.

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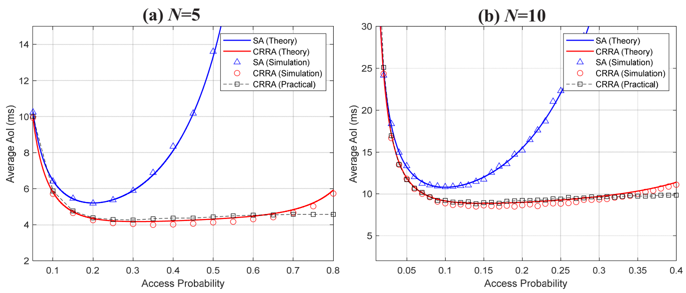


Fig. 5: Average AoI comparison between SA and CRRA: the access probability on the $x$-axis is $p_{SA}$ in SA and $p_{CR}$ in CRRA. The number of users $N$ is (a) 5 and (b) 10. $K_{\max} = 3$ when $K$ is known at the AP. Each update packet has a 256-byte payload.

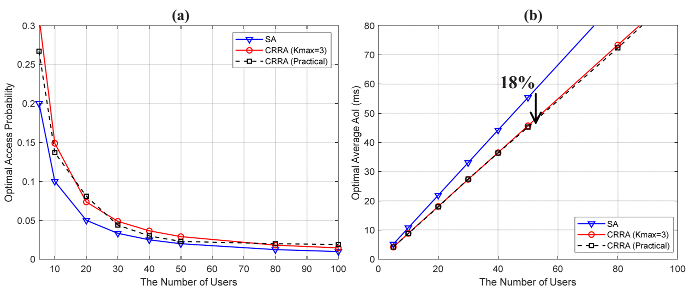


Fig. 6: (a) The optimal access probability and (b) the optimal average AoI versus the number of users. Each update packet has a 256-byte payload.

$N = 10$), as shown in Fig. 5(b). Therefore, CRRA is more robust against the estimation error of the number of users in the random-access system.

We further compare the optimal average AoI between CRRA and SA under different numbers of users in Fig. 6(b). CRRA has a lower optimal average AoI under different $N$, e.g., when the number of users is 50, the optimal average AoI of CRRA is reduced by 18% compared with SA. This indicates that instead of ignoring the collision as in SA, collision resolution in CRRA helps improve information freshness significantly when update packets have a 256-byte payload.

We now evaluate the impact of different payload sizes. Fig. 7 plots the optimal average AoI versus the payload size of an update packet, when the number of users is 30. When the payload size of an update packet is large, CRRA has a significantly lower optimal average AoI compared with SA, because the duration of a polling/trigger frame is negligible compared with an update packet. For example, when the payload size is 256 bytes, the duration of an update packet is around $0.41ms$, while the duration of a polling or trigger frame is only $0.052ms$. We also notice that when the payload size is as small as 32 bytes (i.e., a very tiny packet with a duration of $0.11ms$), the average AoI of CRRA is a little higher than that of SA. To conclude, the larger the payload size, the more significant the improvement of CRRA over SA, as indicated by Fig. 7.

## V. Conclusion

We have investigated an AoI-oriented CRRA protocol for information update systems. Since SA ignores the collision

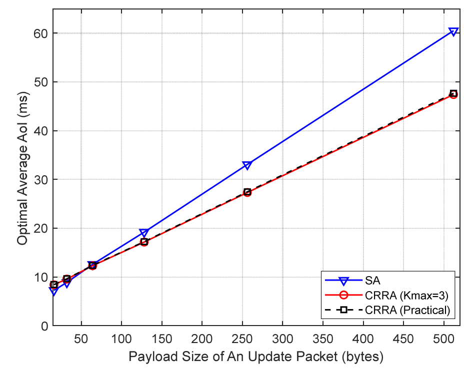


Fig. 7: The optimal average AoI versus the payload size of an update packet. The number of users is 30.

and users simply contend for the channel again, it has a high collision probability in every time slot, resulting in high average AoI. CRRA argues that resolving collisions can significantly improve the information freshness, because only the collided users enter a collision resolution procedure (CRP) (hence having a lower collision probability in the CRP) and the receiver ensures their successful update first. Although the CRP complicates the AoI analysis, we derive the theoretical average AoI of CRRA by assuming the number of collided users $K$ is known at the receiver, which is shown to approximate the practical protocol well, where $K$ is unknown. Simulations show that CRRA outperforms SA by providing a low and stable average AoI, indicating that CRRA is a viable solution for networks with a time-varying number of users.

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